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A11104 119796

NATIONAL INSTITUTE OF STANDARDS &
TECHNOLOGY
Research Information Center
Gaithersburg, MD 20899

A DIRECT-READING DEVICE FOR USE IN COMPUTING CHARACTERISTICS OF VACUUM TUNGSTEN LAMPS (WITH COMPUTING CHART)

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I. INTRODUCTION

Previous work in the derivation of characteristic equations of vacuum tungsten lamps[1] has led to the consideration of the possibility of designing some form of computing device, the construction of which would depend directly upon these equations, and which would yield values of acceptable precision, when compared with those obtained by use of the tables[2] which have been computed from the equations.

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[1] Middlekauff and Skogland, "Characteristic equations of tungsten lamps and their application in heterochromatic photometry," this bulletin, **11**, pp. 483–534; 1914. (Scientific Paper No. 238.)

[2] Pp. 527–534.

The characteristic equations here mentioned are employed in the determination from observed values of voltage, candlepower, and watts per candle the values of these variables at other points, thus obtaining values whose precision is as high as that of the observations used as the basis of the computations. Observations may thus be made at any convenient point; for example, at a voltage corresponding to a match in color with standards whose values are well known, and direct measurements at other points are eliminated. The advantage of measurements at color match may thus always be realized, and the disadvantages of color difference measurements (notably, large deviations of individual observations from their mean) may be eliminated. By this method the following steps are taken in obtaining a solution:

1. Factors read from a reduction table are applied to the observed values to obtain the so-called "normal" values; that is, values at normal wpc (1.20).
2. Voltage ratio is computed, using the normal value as a base.
3. The proper table is referred to at the point corresponding to the voltage ratio, and actual wpc and per cent cp are read.
4. Cp ratio is applied to normal cp to secure the value desired.

In case the voltage corresponding to a certain cp is desired, the second step is the computation of cp ratio, using the normal value as a base; this cp ratio is then the point of reference in the tables. This process, though simple, requires considerable time before a solution is obtained.

The device has passed through the experimental stage, improvements being made as suggested by considerations of convenience to the user. The result finally accomplished is the production of a chart of scales which evaluate any one of the variables—candlepower, watts per candle, or voltage—with a single setting to observed values at any wpc within the range of the device.

II. SCALES OF THE DEVICE

The scales, shown on the plate at the end of this paper, include horizontal scales of volts and wpc. These scales are found at the lower part of the plate. The horizontal lines which are ruled across the cp chart correspond to observed or rated wpc, and they fix the point at which the volt scale is set in computing per cent cp.

The volt scale should be cut from the chart and preferably mounted on a strip of dry hardwood or on metal, care being exercised to avoid shrinkage.[3] Shellacking of any of the scales is not recommended.

The remainder of the chart may be placed on a drawing board or other plane surface.

III. HOW THE SCALES ARE USED

Observed voltage is set to the 100 per cent cp line at its intersection with the horizontal corresponding to observed wpc. Extreme care need not be exercised in interpolating wpc between scale divisions in this setting, but the volt scale should be exactly horizontal. The 100 per cent cp line (hereinafter called the initial cp line) has been drawn as a broken line, so that it is always easily found. From the setting made, per cent cp at any voltage may now be read at the point of intersection with the volt scale corresponding to the voltage at which a value is desired, or the converse problem may be solved by the same setting. For accurate interpolation on the cp chart a scale like that of Fig. 1 may be used. Its use is too obvious to require comment. In practice, this scale is copied on transparent material, such as celluloid or tracing cloth. A scale of this kind was used in obtaining values given in the examples of the following section.

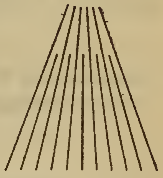

FIG. 1.—*Interpolating scale*

If the volt scale has been set exactly to observed wpc on the initial cp line, wpc at any point desired may be read by dropping a perpendicular to the horizontal wpc scale; but since, as stated above, extreme accuracy is not required in setting to observed wpc on the cp chart, a better method is to apply the volt scale directly to the horizontal wpc scale, placing observed voltage opposite observed wpc. On this wpc scale interpolation may be easily and accurately made.

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[3] If only the volt and wpc scales are to be generally used, both may be detached from the chart and mounted in any convenient slide-rule form, the cp chart still being available for occasional computations

A few examples illustrating different settings of the scales will now be given.

IV. EXAMPLES ILLUSTRATING SCALE SETTINGS AND THEIR PRECISION

In the following examples values read from the scales are compared with those obtained by use of Tables 20 and 22.[4]

Example 1.—Per cent cp and wpc read with 100 volts set to the initial cp line and to 1.20 wpc. From this setting per cent cp and wpc at other volt scale divisions are read. This is evidently a percentage setting of the volt scale. The results are given in Table 1.

TABLE 1

Comparison of Values of Per Cent cp and of wpc at Various Percentages of the Voltage Corresponding to 1.20 wpc, as Read from the Scales and from the Tables

Per cent voltage	Per cent candlepower			Watts per candle		
	Scale	Table 20	Difference	Scale	Table 22	Difference
			Per cent			Per cent
94.0	80.0	79.94	0.08	1.360	1.361	0.07
97.5	91.4	91.29	.12	1.262	1.263	.08
103.0	111.3	111.2	.09	1.131	1.131	.00
107.5	129.4	129.4	.00	1.040	1.040	.00
112.5	151.9	151.8	.07	.953	.9525	.05
117.0	174.0	174.0	.00	.884	.8840	.00
120.0	190.0	189.9	.05	.843	.8431	.01
125.5	221.4	221.4	.00	.776	.7762	.03
128.0	237.0	236.7	.13	.749	.7487	.04
132.0	262.8	262.6	.08	.708	.7084	.06
Mean			.06			.03

Example 2.—Readings of per cent cp and wpc made from setting observed, or rated, values at 1 wpc, and comparison with values computed by the tables.

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[4] Middlekauff and Skogland, loc. cit., pp. 532, 534.

TABLE 2

Volts		Per cent candlepower			Watts per candle		
From observed or rated	To scale	Scale	Table 20	Difference	Scale	Table 22	Difference
				Per cent			Per cent
105	111	121.4	121.4	0.00	0.899	0.899	0.00
113	121	127.0	127.0	.00	.878	.878	.00
115	124	130.0	130.1	.08	.866	.866	.00
117	107	72.9	72.8	.14	1.192	1.193	.08
117	125	126.2	126.0	.16	.882	.881	.11
118	111	80.7	80.6	.12	1.127	1.127	.00
120	110	73.5	73.4	.14	1.187	1.187	.00
Mean				0.09			0.03

EXAMPLE 3.—Observations: 110 volts, 25 candles, 1.35 wpc. Required: (*a*) Cp and wpc at 125 volts; (*b*) volts and wpc for 20 candles.

Solution.—(*a*) Set 110 volts to the initial cp line at its intersection with the horizontal through 1.35 wpc. Read at 125 volts, 158 per cent cp. $1.580 \times 25 = 39.50$ candles, the required value. A similar setting to the horizontal wpc scale gives 1.045 wpc.

(*b*) $\frac{20}{25} = 0.80 = 80$ per cent cp. Set the scales as in (*a*) above and read at 80 per cent cp, 103.5 volts, the required value for 20 candles. Set 110 volts to 1.35 on the wpc scale, and opposite 103.5 volts read 1.532 wpc.

The solution by the much longer method employing Tables 19, 20, and 22 is (*a*) 39.48 candles, 1.045 wpc; (*b*) 103.5 volts, 1.532 wpc.

Summary of results.—(1) Scale values: (*a*) 39.50 candles, 1.045 wpc; (*b*) 103.5 volts, 1.532 wpc.

(2) Table values: (*a*) 39.48 candles, 1.045 wpc; (*b*) 103.5 volts, 1.532 wpc.

EXAMPLE 4.—Observations: 110 volts, 88 candles, 1.05 wpc. Required: Volts and candles at 0.7 wpc.

Solution.—Set 110 volts to 1.05 wpc. Read at 0.7 wpc, 136.6 volts, the required value. Set 110 volts to the initial cp line on the horizontal through 1.05 wpc. Read at 136.6 volts, 211.2 per cent cp. 88 × 2.112 = 185.9 candles, the value required. This solution is identical with that obtained by employing Table 19.[5]

EXAMPLE 5.—The following example, which has been given in part in Table 8, page 506, of the paper above referred to (note 1), illustrates the application of the scales in the adjustment of several observations. Three solutions by the same method are given, so that the scales (the cp scales in particular) may be tested on several wpc lines.

This method is somewhat different from that given in the discussion of characteristic equations, but, although not quite so precise, it results in securing very nearly the same adjustment of values. It is based on the validity of the arithmetical mean and therefore appears to be a sound method. One observed value is selected as a base for scale settings. After reading from the scales the values at each voltage, these values are adjusted to give the same mean as the observed values. This is accomplished by increasing or decreasing each observed value by the same percentage as that required to correct the mean of the observed values to the mean of the computed values, a procedure familiar in other applications to all who have had laboratory experience in any line.

Results and comparisons are given in Table 3.

TABLE 3

Adjustment of observed cp Values of a Group of Seven 40-watt Drawn Wire Lamps

Observations				Scale values applied as factors to—			Adjusted values, from mean based on—		
Number	Volts	Candles	(wpc)	Obs. 1	Obs. 4	Obs. 7	Obs. 1	Obs. 4	Obs. 7
1	96	18.20	(1.633)	18.20	18.33	18.18	18.27	18.30	18.24
2	104	24.48	....	24.43	24.58	24.46	24.52	24.54	24.54
3	112	32.15	....	31.94	32.04	31.98	32.06	31.98	32.09
4	120	41.07	(1.032)	40.82	41.07	40.89	40.97	41.00	41.03
5	128	51.46	....	51.25	51.55	51.32	51.44	51.46	51.50
6	136	63.48	....	63.16	63.45	62.96	63.39	63.33	63.18
7	144	76.71	(0.736)	76.66	77.10	76.71	76.94	76.96	76.97
Mean	....	43.94	....	43.78	44.02	43.79	43.94	43.94	43.94

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[5] Middlekauff and Skogland, loc. cit. This example is given on p. 524.

TABLE 3—Continued

Mean of adjusted values	Values obtained from Tables 19 and 20
18. 27	18. 26
24. 53	24. 51
32. 04	32. 05
41. 00	40. 97
51. 47	51. 37
63. 30	63. 33
76. 96	76. 91
43. 94	43. 91

Maximum difference, 0.19 per cent.
Average difference, 0.07 per cent.

The differences which appear in these examples between values read from the scales and those computed by employing Tables 19, 20, and 22 are certainly so small that no valid objection can be raised against the use of the device as here presented without raising a similar objection to the tables of the previous paper; for the observed values are checked very nearly as closely by one method as by the other. Such differences as do occur may be attributed to slight errors in construction, width of line of the original drawing necessary to secure a plain reproduction, and errors in interpolation.

V. ADVANTAGES AND DISADVANTAGES

When compared with the method involving use of Tables 19 to 22, the great advantage that is first apparent in favor of this device is a saving of time, a fact which will be evident to any who solve the examples of the preceding section by each method.

This results because of the following facts, namely, reduction to normal wpc is eliminated; direct and converse problems may be solved with equal ease; voltage ratios need not be computed; the volt and wpc scales constitute a complete slide rule from which, for example, life-test voltages may be read directly for any wpc, or the wpc at the nearest rack voltage may be read from the same setting.

An advantage over any method employing exponents is that use of the exponential rule or of logarithm tables is unnecessary. This method, in common with the exponential method, avoids reduction to normal wpc. In the exponential method, however, several charts of curves, constructed as previously described,[6] are necessary in solving both direct and converse problems. For example, curves giving exponents which, when applied to voltage ratio, evaluate cp ratio can not be employed in solving the converse problem. Further, voltage and wpc ratios must be known before the exponential method can be used.

There are but few disadvantages, of which the following may be mentioned: A device which would include the whole range in wpc covered by the tables would either be inconveniently large, or, if reduced to a convenient size, there would be some sacrifice of precision; the values obtained even from scales perfectly constructed are modified by personal errors in interpolation between scale divisions. These errors may, however, be greatly reduced by using the interpolating scale shown in Fig. 1.

VI. CONSTRUCTION OF THE SCALES

1. RANGE SELECTED

In order to minimize the disadvantages just mentioned, the scales have been designed to include a range in wpc of from 0.70 to 2.05, with a volt scale extending from 94 to 166 volts, these limits being determined by the size of section-ruled paper available, with due consideration of the voltages and efficiencies most generally employed in laboratory and factory practice. It is believed that the range of these scales is sufficient to include settings and solutions for 105 to 130 volt lamps in standardizing and life-test problems.

A full description of the design of these scales will now be given. The method outlined may be used in the construction of scales of any range and is general in its application to functions which may be expressed by related equations of the same form as those expressing the characteristics here considered.[7] By extension of the method, all similarly related quadratic equations involving either x or y as independent variable, if these variables are of exponential significance, may be similarly represented after the proper choice of coordinates and may be used in exactly the same way as are the scales here designed.

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[6] Middlekauff and Skogland, loc. cit., p. 513.

[7] For characteristic equations, see pp. 496, 498.

2. COORDINATES TO WHICH ALL POINTS ARE REFERRED

All of the scales are based upon scales of equal parts corresponding to logarithmic per cent voltage, which may be considered as a whole number (for convenience, it is here so considered) or as a ratio.

From reciprocals of values of "Volts" given in Table 19,[8] which contains factors for reduction to 1.20 wpc, it was found that a range in wpc of from 0.70 to 2.05 includes values of per cent voltage of from 76 to 133, approximately. The corresponding logarithms are included within the range 1.880 to 2.130. These numbers, 1.880 to 2.130, were indicated in pencil in a horizontal direction as the X scale of equal parts. For reasons which will appear later, the same scale was repeated in a vertical direction as a Y scale. Due to limitations imposed by the width of the plotting paper, the total length of the Y scale was made one-half that of the X scale. Any practical value may be assigned to the ratio of these scales.

3. WATTS PER CANDLE SCALES

From reciprocals of values of "Volts" given in Table 19 [8] the horizontal wpc scale was constructed in steps of 0.01, each point being placed at the corresponding value of logarithmic per cent voltage. The same scale was plotted vertically in steps of 0.05 wpc from 2.05 to 1 and of 0.02 wpc for the remainder of the scale. The last-named scale divisions were extended horizontally across the chart, as shown on the plate. On drawings of smaller range the Y scale may be made equal in length and in number of scale divisions to the X scale, but this is a matter of no great importance since, as stated in Section III above, extreme accuracy in interpolation on this scale is not required.

These wpc scales are not logarithmic,[9] as they are based upon a second-degree logarithmic equation.[10]

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[8] Middlekauff and Skogland, loc. cit., p. 527. See also p. 500, sec. 6, lines 19–23.

[9] "Logarithmic"; that is, two such scales would not form scales A and B of the ordinary slide rule.

[10] Middlekauff and Skogland, loc. cit., p. 496, eq. 1a.

4. NORMAL PER CENT CANDLEPOWER SCALE

From Table 20[11] may be read per cent cp for any voltage ratio on the normal basis (1.20 wpc), so that tabulated values may be plotted in the same manner as was done in constructing the wpc scales; that is, steps of per cent cp may be plotted opposite the corresponding logarithmic per cent voltage, this construction giving a normal per cent cp scale. Assume this scale drawn.

5. PRELIMINARY PER CENT VOLTAGE SCALE

If, now, a per cent voltage scale be drawn in with points placed at the corresponding logarithms X, there is completed a set of scales from which values of per cent cp and of wpc may be read for any percentage of normal voltage; that is, the tabulated values have been transferred to the scales, and the device is, as yet, no improvement over the tables, because it could at best yield only the tabulated values, with little, if any, saving of time. It need hardly be stated that the present choice of coordinates would not have been necessary if this were the final result to be achieved. The purpose of the choice made becomes evident in what follows.

6. PRELIMINARY VOLT SCALE

If, now, the per cent voltage scale just constructed be free to move—that is, detached from the chart—it may be used as a volt scale of range from 76 to 133 volts, which is a considerable improvement, as voltage ratios need not be computed. Obviously, this transformation of the per cent voltage scale to a volt scale follows directly from the fact that the scale is logarithmic, and hence is, by construction, a computer of voltage ratio for any setting.

7. USE OF THE SCALES CONSTRUCTED ABOVE

These three scales (wpc, volts, and normal per cent cp) may be used in connection with reduction Table 19 (see note 2), normal cp only being computed by applying to the observed value the factor (F_0) read from the table, or the reciprocal (F_1) of this factor may be read from the normal per cent cp scale at the observed voltage, which is set to observed wpc. At the required voltage is read the cp factor (F_2) on the normal basis and the actual wpc. If P_0 is the observed value in candles, and P_1 is the required value, $P_1 = P_0 F_0 F_2 = \frac{P_0}{F_1} F_2$. It is to be noted that the reduction $P_0 F_0 = \frac{P_0}{F_1}$ is a reduction of observed cp to normal cp, but that the corresponding normal voltage need not be read. There then remains only the reduction to normal cp to dispense with before being free from use of the tables or from taking two readings from a normal per cent cp scale. This is accomplished by the construction of the per cent cp curves, which, as will be proven, are straight lines.

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[11] Middlekauff and Skogland, loc. cit., p. 532.

8. PER CENT CANDLEPOWER CURVES

As the second characteristic equation (see note 7) is employed in proving some of the relations which are derived below, it is here quoted.

This equation is:

$$y_2 = - .946x^2 + 3.592x \qquad (1)$$

in which $y_2 = \log$ cp ratio and $x = \log$ voltage ratio, the conditional relation between x and y_2 being that $x = y_2 = \log 1 = 0$ at actual wpc $= 1.20$ (the normal value).

From equation 1 is derived by differentiation,

$$\frac{dy_2}{dx} = -1.892x + 3.592, \qquad (2)$$

which shows that $\frac{dy_2}{dx}$ decreases as x increases above its normal value. Since wpc and consequently logarithm wpc also decrease, it follows that at wpc less than 1.20 the change of logarithm cp ratio with logarithm voltage ratio, and hence of cp ratio with voltage ratio, is less than at the normal value of 1.20 wpc; hence the distance in scale divisions from 100 per cent to any other per cent cp on a scale constructed in the manner given in paragraph 4, page 278, but based on a wpc less than 1.20, would be greater than on the normal scale. By similar reasoning, since x is negative for voltage ratios less than 1, this distance would be less at wpc greater than 1.20. For example, a cp scale constructed with its

100 per cent point at 0.8 wpc will contain fewer scale divisions for a given range in wpc than will a similar scale constructed at 1.6 wpc.

Suppose, now, that the normal cp scale constructed above be moved up to the 1.20 line of the Y wpc scale constructed in paragraph 3, page 277. It will evidently bear the right relation to all of the scales so far constructed. Also, the 100 per cent cp point for any wpc may be traced as a straight line drawn through corresponding points of the horizontal and vertical wpc scales. By construction this line also passes through corresponding values of logarithmic per cent voltage. This line is then the first of the cp curves and is designated as the initial cp line. If other cp scales be constructed at points above and below the normal scale just placed, values being obtained from the tables, a series of curves may be drawn through corresponding points of the different scales. These curves are straight lines, which greatly simplifies their construction and makes it evident why the vertical scale was based on a logarithmic scale of equal parts. That these curves are straight lines may be proven as follows: Equation (1) above may be written

$$Y_2 = X^{-.946x+3.592} = X^{u_2} \tag{3}$$

in which Y_2 = cp ratio and X = voltage ratio, u_2 being a variable function of x expressed by the equation

$$u_2 = -.946x + 3.592, \tag{4}$$

from which is obtained by differentiation

$$du_2 = -.946dx, \tag{5}$$

or, expressed in words, u_2 changes uniformly with x.

It therefore follows that the curve for any per cent cp must be displaced from the initial cp line by a distance which is constantly changing with x, since an increase in exponent corresponds to an increase in distance on a logarithmic scale. To illustrate: Log 8 = log 2^3 = 3 log 2. If log 2 be plotted as one scale division, log 8 would be found at three scale divisions; the exponent is increased from 1 to 3 and the distance is increased in like degree. These curves are therefore straight lines which, as shown above, diverge from the initial cp line in passing to the lower wpc.

So that in construction two points only need be accurately placed for each line, preferably one above and one below the normal scale, throughout its range, and either two or three points for the lines which do not cross the normal scale. A third point in any case is a check on the accuracy of placing the other two, and is not absolutely necessary.

9. FINAL VOLT SCALE

The volt scale designed in sections 5 and 6, page 278, might be used, but a scale of different range constructed, however, in exactly the same manner is given on the plate. Logarithms corresponding to the voltages chosen (94 to 166) were substituted for the original per cent voltage logarithms of the X scale, the same distance between successive points being of necessity adhered to. The volt and half-volt divisions were drawn in at points corresponding to their logarithms. Any volt scale desired may be constructed in a similar manner.

The actual values through which the lines of the chart were drawn are given in Section VIII, so that the entire chart or any section of it may be drawn to any scale desired.

VII. THE COMPUTER IN A MODIFIED FORM

From these data it is possible to construct a circular or disk computer, although the construction presents points of considerable difficulty. Volt and wpc scales may be easily so arranged, and their construction is not difficult.

Since, however, the cp curves of a circular computer would be drawn through points determined by radial and circumferential voltage logarithms, they would no longer be straight lines, and more than three points would have to be placed for each curve. As a result their construction would be rather difficult.

VIII. TABLES OF VALUES USED IN CONSTRUCTING THE SCALES

Scale divisions are here considered as whole numbers; e. g., log 94 is given instead of log 0.94 as the first point of the volt scale. Rectangular cartesian coordinates are used and *X* and *Y* have their usual graphical significance.

1. VOLT SCALE

The values of *X* for each scale division on a reference scale of equal parts are given in Table 4. This is the only logarithmic scale of the device.

TABLE 4

Values of X Corresponding to Values of Voltage Indicated by Scale Divisions

Scale	X	Scale	X	Scale	X	Scale	X
94.0	1.97312	110.5	2.04336	127.0	2.10380	143.5	2.15685
94.5	543	111.0	532	127.5	551	144.0	836
95.0	772	111.5	727	128.0	721	144.5	986
95.5	1.98000	112.0	921	128.5	890	145.0	2.16136
96.0	227	[12] 112.5	2.05115	129.0	2.11058	145.5	286
96.5	452	113.0	307	129.5	226	146.0	435
97.0	677	113.5	499	130.0	394	146.5	583
97.5	900	114.0	690	130.5	561	147.0	731
98.0	1.99122	114.5	880	131.0	727	147.5	879
98.5	343	115.0	2.06069	131.5	892	148.0	2.17026
99.0	563	115.5	258	132.0	2.12057	148.5	172
99.5	782	116.0	445	132.5	221	149.0	318
100.0	2.00000	116.5	632	133.0	385	149.5	464
100.5	216	117.0	818	133.5	548	150.0	609
101.0	432	117.5	2.07003	134.0	710	151	897
101.5	646	118.0	188	134.5	872	152	2.18184
102.0	860	118.5	371	135.0	2.13033	153	469
102.5	2.01072	119.0	554	135.5	193	154	752
103.0	283	119.5	736	136.0	353	155	2.19033
103.5	494	120.0	918	136.5	513	156	312
104.0	703	120.5	2.08098	137.0	672	157	589
104.5	911	121.0	278	137.5	830	158	865
105.0	2.02118	121.5	457	138.0	987	159	2.20139
105.5	325	122.0	635	138.5	2.14144	160	412
106.0	530	122.5	813	139.0	301	161	682
106.5	734	123.0	990	139.5	457	162	951
107.0	938	123.5	2.09166	140.0	612	163	2.21218
107.5	2.03140	124.0	342	140.5	767	164	484
108.0	342	124.5	516	141.0	921	165	748
108.5	542	125.0	691	141.5	2.15075	166	2.22010
109.0	742	125.5	864	142.0	228		
109.5	941	126.0	2.10037	142.5	381		
110.0	2.04139	126.5	209	143.0	533		

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[12] Example: Log 112.5=2.05115, the point on the *X* scale of equal parts at which the 112.5 volt division is placed.

2. WATTS PER CANDLE SCALE

In Table 5 are given values of X corresponding to wpc scale divisions. These values of X are in part repeated as ordinates Y, through which horizontals are drawn across the per cent cp chart. All of these values may be repeated for placing horizontals, if it is so desired. The values given are based on a reference scale of equal parts. The length of the Y scale may bear any convenient ratio to that of the X scale.

TABLE 5

Values of X Corresponding to Values of wpc Indicated by Scale Divisions

Scale	X	Scale	X	Scale	X	Scale	X
0.70	2.12346	1.04	2.03138	[13] 1.38	1.97021	1.72	1.92478
.71	003	.05	2.02926	.39	1.96869	.73	361
.72	2.11663	.06	715	.40	718	.74	245
.73	332	.07	507	.41	569	.75	129
.74	002	.08	301	.42	421	.76	013
.75	2.10682	.09	098	.43	273	.77	1.91900
.76	363	.10	2.01897	.44	126	.78	786
.77	053	.11	701	.45	1.95983	.79	673
.78	2.09745	.12	501	.46	841	.80	560
.79	444	.13	308	.47	699	.81	450
.80	146	.14	114	.48	558	.82	339
.81	2.08854	.15	2.00924	.49	419	.83	229
.82	565	.16	735	.50	280	.84	119
.83	283	.17	549	.51	142	.85	013
.84	002	.18	364	.52	005	.86	1.90904
.85	2.07727	.19	181	.53	1.94869	.87	796
.86	455	.20	000	.54	734	.88	689
.87	188	.21	1.99822	.55	600	.89	584
.88	2.06923	.22	645	.56	466	.90	480
.89	663	.23	465	.57	336	.91	375
.90	405	.24	294	.58	207	.92	271
.91	153	.25	121	.59	078	.93	169
.92	2.05902	.26	1.98949	.60	1.93949	.94	066
.93	656	.27	782	.61	823	.95	1.89965
.94	412	.28	615	.62	697	.96	863
.95	172	.29	449	.63	571	.97	763
.96	2.04934	.30	284	.64	446	.98	664
.97	701	.31	122	.65	323	.99	565
.98	469	.32	1.97960	.66	200	2.00	466
.99	241	.33	800	.67	078	.01	367
1.00	014	.34	642	.68	1.92956	.02	272
.01	2.03792	.35	484	.69	834	.03	177
.02	571	.36	326	.70	713	.04	079
.03	354	.37	173	.71	595	.05	1.88981

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[13] Example: In Table 19 at 1.38 wpc find 107.1 "Volts." Log $\frac{100}{1.071}$=1.97021, the point on the X scale of equal parts at which the 1.38 wpc division is placed.

In computing tables similar to Table 5 above, values of X for every scale division need not be determined as given in the example. Values of sufficient accuracy may be obtained by computing X for wpc points in steps of 0.12. These values are then differenced to the second order, interpolated twice to halves and once to thirds, thus completing the table except the points near the beginning and end, which may be interpolated by differencing to the fifth order and employing Newton's or Stirling's interpolating coefficients. Rules for these interpolations may be found in any text on the subject.

3. PER CENT CANDLEPOWER CHART

The curves of this chart are straight lines. Although two points accurately placed determine a straight line, three points are given for the longer lines, so that errors due to slight displacement of points and to possible inaccuracies of a straight edge used in ruling in the lines, are minimized.

All of these lines are referred to the horizontal and vertical scales of equal parts, the points chosen falling on even divisions of the vertical scale, so that possible errors due to interpolation will occur only in the X direction. These errors are minimized by choosing a long X scale. The construction must be very carefully carried out to place these lines in proper relation to the other scales. Values for each line of this chart are given in Table 6.

TABLE 6

Values of X Corresponding to Values of Y

[A straight line drawn through the points given in the body of the table places the per cent cp line corresponding to the scale division indicated in the first column of the table]

Scale	Values of X			Scale	Values of X		
		$Y=2.110$	$Y=2.120$			$Y=2.100$	$Y=2.120$
15.0		1.88117	1.89001	17.0		1.88653	1.90437
15.5		1.88489	1.89376	17.5		1.88982	1.90771
16.0		1.88851	1.89742	18.0		1.89305	1.91096
16.5		1.89204	1.90093	18.5		1.89618	1.91414
				19.0		1.89924	1.91721
				19.5		1.90223	1.92025

TABLE 6—Continued

Values of X Corresponding to Values of Y—Continued

Scale	Values of X		
		$Y=2.080$	$Y=2.100$
20	....	1.88708	1.90514
21	....	1.89266	1.91076
22	....	1.89800	1.91614
23	....	1.90311	1.92129
24	....	1.90800	1.92624
25	....	1.91271	1.93100
26	....	1.91724	1.93559
27	....	1.92162	1.94001
		$Y=2.040$	$Y=2.100$
28	....	1.88891	1.94429
29	....	1.89293	1.94840
30	....	1.89682	1.95239
31	....	1.90057	1.95627
32	....	1.90423	1.96002
33	....	1.90778	1.96367
34	....	1.91122	1.96721
35	....	1.91458	1.97066
36	....	1.91783	1.97402
37	....	1.92101	1.97729
38	....	1.92412	1.98048
39	....	1.92712	1.98359
	$Y=1.980$	$Y=2.000$	$Y=2.100$
40	....	1.89227	1.98664
42	....	1.89786	1.99249
44	1.88416	1.90321	1.99810
46	1.88923	1.90832	2.00346
48	1.89410	1.91324	2.00864
50	1.89879	1.91796	2.01360
52	1.90331	1.92251	2.01837
	$Y=1.960$	$Y=2.000$	$Y=2.100$
54[14]	1.88835	1.92689	2.02301
56	1.89252	1.93113	2.02755
58	1.89653	1.93524	2.03177
60	1.90042	1.93921	2.03595
62	1.90420	1.94304	2.03997

Scale	Values of X		
	$Y=1.940$	$Y=2.000$	$Y=2.100$
64	1.88836	1.94677	2.04391
66	1.89189	1.95040	2.04774
68	1.89530	1.95393	2.05142
70	1.89863	1.95734	2.05503
72	1.90187	1.96068	2.05854
74	1.90501	1.96394	2.06198
76	1.90810	1.96709	2.06532
78	1.91108	1.97018	2.06856
	$Y=1.920$	$Y=2.000$	$Y=2.100$
80	1.89427	1.97321	2.07173
85	1.90128	1.98044	2.07936
90	1.90781	1.98730	2.08661
95	1.91406	1.99380	2.09346
100	1.92000	2.00000	2.10000
	$Y=1.890$	$Y=2.000$	$Y=2.100$
105	1.89559	2.00590	2.10623
110	1.90092	2.01155	2.11220
115	1.90604	2.01697	2.11792
120	1.91095	2.02216	2.12342
	$Y=1.890$	$Y=2.000$	$Y=2.050$
125	1.91567	2.02716	2.07791
130	1.92022	2.03199	2.08287
135	1.92462	2.03664	2.08764
140	1.92885	2.04112	2.09225
145	1.93293	2.04547	2.09673
150	1.93689	2.04967	2.10105
155	1.94074	2.05373	2.10524
160	1.94447	2.05770	2.10931
165	1.94808	2.06153	2.11327
170	1.95159	2.06528	2.11711
175	1.95502	2.06889	2.12084
180	1.95835	2.07243	2.12447
185	1.96159	2.07588	....
190	1.96476	2.07925	....
195	1.96785	2.08253	....

[14] Example: Number corresponding to 1.960 is 91.20 per cent voltage. In Table 20 find at 91.20 "volts" 71.59 "cp." 54 per cent of 71.59=38.66. In Table 20 find 77.33 "volts" corresponding to 38.66 "cp." Log 77.33=1.88835, a point on the X scale of equal parts and in the 54 per cent cp line.

TABLE 6—Continued

Values of X Corresponding to Values of Y

Scale	Values of X		
	$Y=1.890$	$Y=2.000$	$Y=2.050$
200	1.97085	2.08572	....
210	1.97668	2.09191	....
220	1.98225	2.09785	....
230	1.98757	2.10353	....
240	1.99269	2.10897	....
250	1.99761	2.11421	....
260	2.00234	2.11926	....
270	2.00694	2.12415	....
	$Y=1.890$	$Y=1.980$	
280	2.01136	2.10741	....
290	2.01561	2.11193	....
300	2.01974	2.11628	....
310	2.02375	2.12051	....
320	2.02764	2.12463	....
	$Y=1.890$	$Y=1.960$	
330	2.03145	2.10694	....
340	2.03511	2.11076	....
350	2.03870	2.11451	....
360	2.04218	2.11813	....
370	2.04556	2.12166	....
380	2.04887	2.12509	....

Scale	Values of X		
	$Y=1.890$	$Y=1.940$	
390	2.05212	2.10660	....
400	2.05526	2.10985	....
420	2.06134	2.11611	....
440	2.06715	2.12212	....
	$Y=1.890$	$Y=1.920$	
460	2.07273	2.10575	....
480	2.07809	2.11123	....
500	2.08325	2.11647	....
520	2.08821	2.12153	....
540	2.09300	2.12642	....
	$Y=1.890$	$Y=1.910$	
560	2.09764	2.11995	....
580	2.10209	2.12450	....
	$Y=1.870$	$Y=1.890$	
600	2.08404	2.10643	....
620	2.08817	2.11066	....
640	2.09223	2.11474	....
660	2.09614	2.11869	....
680	2.09996	2.12254	....

IX. SUMMARY

As described above, the characteristic equations of vacuum tungsten lamps, derived and discussed in a previous paper (see note 1), have been employed in constructing a computing device consisting of volt, wpc, and per cent candlepower scales, from which may be read, by a single setting of the movable volt scale to observed values within the range of from 0.70 to 2.05 wpc, values of any one of the variables at other points. The volt scale has a range of from 94 to 166 volts. The limits in wpc and voltage were chosen to include values most generally used in laboratory and factory practice.

A test of the device, illustrated in part by examples given in this paper, indicates that, on an average, values of per cent cp and of wpc read therefrom deviate from those obtained by use of Tables 20 and 22 (see note 2, p. 269) by amounts not exceeding 0.10 per cent and 0.05 per cent, respectively; also that values read from the scales do, on an average, check observed values very nearly as closely as those computed by use of the tables. Such differences as occur when compared with values computed by the tables may be traced to slight errors in construction, width of lines, and modification of values by personal error in interpolation. Differences due to this last cause may be minimized by using an interpolating scale. In scales of smaller range the interpolating scale may be dispensed with, as a greater number of scale divisions may be drawn for a given range; for example, the candlepower chart may be constructed in steps of 1 per cent.

The chief advantage which this device possesses over other methods of characteristic evaluation, including the use of the tables referred to above, is a great saving of time, this resulting because of the directness of solutions without reference to normal wpc, voltage ratios, exponents, etc.

All of the points (referred to horizontal and vertical scales of equal parts) through which lines of the scales of this device were drawn have been tabulated, so that similar scales of the same or different range may be constructed directly from these values.

Sufficient description of the derivation of these values and of the relative position of the scales has been given to permit the construction of scales not included within the wpc range here employed.

Mention has been made of the possibility of constructing this device in circular or disk form, the volt and wpc scales being well adapted to this construction.

The device as given on the plate is ready for use as soon as the volt scale is cut off from the drawing.

It is hoped that this device will lessen the labor of any who have to compute characteristics of vacuum tungsten lamps and that the general efficiency of testing bureaus where it may be used will be raised by the saving of time, which is its chief merit. The device has been made as simple as possible, and even the novice may use it after a little preliminary practice.

WASHINGTON, April 9, 1915.

INSTRUCTIONS FOR CHART

Remove the scale marked "Volts" from the plate. Place on the dotted diagonal of the candlepower chart (upper part of the plate) at its intersection with the horizontal wpc line nearest that corresponding to the observed value (preferably a value obtained from an observation made at color match with the standards used) the point on the volt scale corresponding to the observed value of voltage. Read on the cp chart the value of per cent cp at any desired value of voltage; or, conversely, the voltage for any per cent cp.

Apply observed voltage to observed wpc (the horizontal scale just below the cp chart) and read the value of wpc at the desired voltage; or, conversely, read the voltage for any desired wpc.

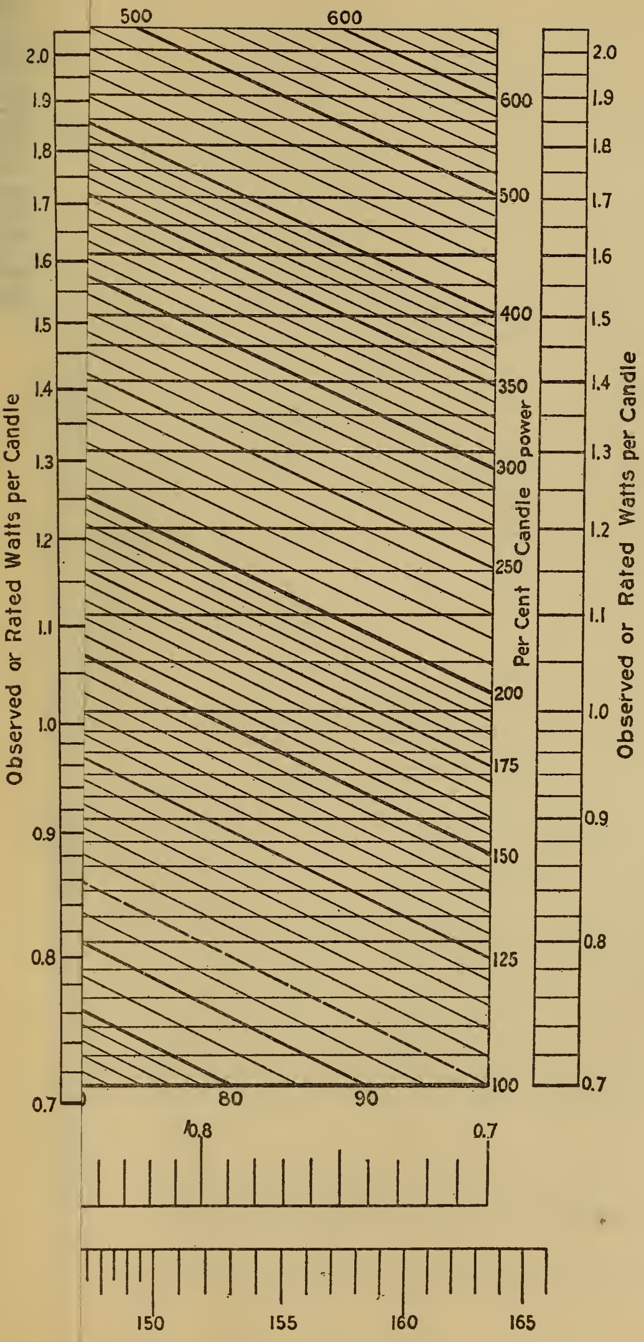
500
600
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
Observed or Rated Watts per Candle
600
500
400
350
300
250
200
175
150
125
100
Per Cent Candle power
80
90
0.8
0.7
150
155
160
165

Fig. 2.—Scales for Computing Characteristics of Vacuum Tungsten Lamps